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# FJORDENV - A WATER QUALITY MODEL FOR FJORDS AND OTHER INSHORE WATERS

Anders Stigebrandt



Department of Oceanography  
GÖTEBORG 2001



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# **FjordEnv - a water quality model for fjords and other inshore waters**

## **Abstract**

This report presents a model, FjordEnv, which gives quick approximate estimates of environmental conditions including physical circulation in fjords and other inshore areas. The model computes rates of mixing intensity and water exchange and residence times in different depth strata. It also computes the expected rate of oxygen consumption and oxygen minimum in the basin water. Furthermore, the model computes changes of water quality due to changes in the supply of nutrients and organic matter from fish farms and other sources. Water quality is measured as Secchi depth in the surface layer and oxygen conditions in the basin water. FjordEnv has been in use in Norway for about one decade (under the name 'Fjordmiljø').

Before the model is applied to a specific area, information on topography and forcing must be gathered. Some of the forcing is derived from offshore conditions like tidal amplitude, density variations in the water column and the natural vertical flux of organic matter. These vary essentially only on regional and larger scales and this forcing may be stored in a database. It is then easy to apply the model to inshore areas in regions covered by the database. Hydrographic measurements are not needed for the computations but can be used optionally to improve the quality of the computations.

## **1. Introduction**

### **1.1 Background**

At visits to fjords, bays and other inshore areas, humans receive impressions related to vision and smell of the water. If the impressions are liked the water quality may, subjectively, be deemed as high. The scientific approach to the issue of water quality is to define water quality in terms of measurable quantities like optical transparency, out-gassing of smelling substances, concentrations of different species of e.g. phytoplankton, macro algae and fish and even diversity of species. Obviously, water quality of an area of the sea is strongly related to its ecology. It is an important scientific challenge to find out how water quality in a certain area should be defined properly and how it is related to external forcing including anthropogenic contributions.

Increasing fear of possible environmental impact of marine fish farming led to intensified research in the 1980'ies aiming at assessment and prediction of environmental effects of farming in inshore waters. In the so-called Møre project, about 30 fjords in Møre and Romsdal at the West Coast of Norway were investigated, making it possible to study various effects of topography (Aure & Stigebrandt, 1989a, 1990). Together with already existing knowledge, the new knowledge about fjords developed during the Møre project laid the foundation for the development of the water quality model Fjord Environment (FjordEnv, in Norwegian: Fjordmiljø). The model predicts qualitative effects of eutrophication due to fish farming and other human activities in and around fjords and other inshore areas. PC implementations of FjordEnv have been in use for about one decade. A booklet in Norwegian has served as an introduction to the scientific base for the computations (Stigebrandt, 1992).

The present report describes and discusses the general problem of water quality modelling in inshore areas with special reference to the water quality model FjordEnv. This particular model is fairly general and it is believed that it may be used and further developed in research programmes like MARE (MARine Research on Eutrophication – Swedish MISTRA) and OAERRE (Oceanographic Applications to Eutrophication in Regions of Restricted Exchange - EU). The overall goal of MARE is to develop a generally accepted and open user-friendly decision-support system as a tool to develop and test cost-effective strategies to reduce eutrophication in the Baltic Sea. The MARE system will build on a series of models linking information about ecosystem properties, biogeochemical processes, physical transports, nutrient inputs, and costs for nutrient reductions. OAERRE aims to understand the physical, biogeochemical and biological processes, and their interactions, involved in eutrophication in coastal marine Regions of Restricted Exchange (RREs), especially lagoons and fjords. The scientific issues addressed include the controls on horizontal and vertical exchange in RREs and the response of coastal ecosystems to nutrient enrichment. One of the aims of OAERRE is construction of simplified 'screening' models for the definition, assessment and prediction of eutrophication, involving collaboration with 'end-users', and the use of these models to analyse the costs and benefits of amelioration scenarios.

## 1.2 Prerequisites for modelling of inshore water quality

Along vast stretches of coasts, inshore areas with similar morphological structure often have similar water quality, indicating that offshore conditions may play a great role for inshore water quality. The explanation is that inshore areas usually are flushed essentially by offshore water and the rate of flushing of different depth strata is strongly influenced by inshore morphology and offshore forcing. However, there are many examples of local anomalies due to local outlets of substances affecting the ecology. Local supply of plant nutrients is probably the most common cause of changed water quality in inshore areas.

It is reasonable to assume that the ecological response to small changes of nutrient supply is linear and essentially quantitative. If the nutrient supply changes only little, one thus expects that the quantitative production of biomass changes only little with insignificant qualitative changes so that the composition of species remains unchanged. However, large changes of nutrient supplies may give rise to large quantitative changes of the production of organic matter that may lead to qualitative changes of the structure of the ecosystem. Earlier dominating species are maybe replaced by other species better suited to the new situation.

The niche of so-called water quality models is prediction of water quality in response to changes of the forcing, e.g. changes of supplies of inorganic nutrients and organic matter. One may think that a water quality model should be a coupled physical-biogeochemical-ecological model describing physical circulation, nutrient cycling and the detailed ecology of an area. However, such a model would probably be too complicated and expensive to develop and implement and the cost of the field measurement program needed to provide data for initialisation, forcing and verification of the model would be very high. For predictions related to eutrophication, it is common to use simplified models focusing on quantitative aspects of production and decomposition of organic matter. However, it is believed that some qualitative ecological changes may be predicted as well, using empirical relationships between key properties of an area (e.g. different characteristics of salinity, temperature, bulk production of organic matter) and the ecological state. Such empirical relationships, with particular reference to the Baltic Sea, are under development in MARE.

A water quality model for eutrophication effects in inshore areas should thus predict at least quantitative changes of both primary production in the surface layer and oxygen consumption in the basin water in response to changed supplies of nutrients and organic matter. Changes in primary production may be expressed as changes of the so-called Secchi depth. These specific measures of water quality, i.e. changes of the Secchi depth and oxygen conditions in the basin water, are directly relevant to the quantitative aspect of eutrophication and have the advantage of being robust, easy to observe and possible to predict.

The environmental impact on different depth strata of inshore waters by supply of nutrients and organic matter is crucially dependent on the rates of supply and on the residence time of water. The rate of water exchange and the hypsography of the area determine the latter. Accordingly, a water quality model must compute the rate of water exchange of both the surface layer and the basin water and have a good volumetric description of the inshore area. The model should thus

include the generally most important mechanisms of water exchange in inshore areas as well as locally relevant information on topography, hypsography and forcing.

### **1.3 Philosophy behind and major elements of the water quality model FjordEnv**

The water quality model FjordEnv computes changes from known states with regard to Secchi depth and oxygen conditions in the basin water due to changed supplies of inorganic nutrients and organic matter. To compute changes of water quality relative to known states is usually quite relevant with regard to effects of human impact. It is a relatively simple task compared to, for instance, prediction of the ecological state of an area from general principles. However, even to compute changes relative to known states is rather demanding. Some of the major elements of FjordEnv are briefly mentioned below.

Water exchange of inshore areas is considered to be due to three different modes of circulation. These are tidally forced barotropic circulation, estuarine circulation forced by local supply of freshwater and wind mixing and so-called intermediary circulation. The latter two are baroclinic, forced internally (locally) and externally (remotely), respectively. FjordEnv uses the model for estuarine circulation developed by Stigebrandt (1975, 1981). At least in Scandinavian fjords and Baltic Sea bays, the intermediary circulation is usually the dominating mode of water exchange. This mode of water exchange has often been overlooked and is described only in relatively few publications. It is forced remotely by time-dependent offshore variations of the density field i.e. variations in the coastal water outside the inshore area. A formula for the mean rate of water exchange due to intermediary circulation, derived from results of the numerical model in Stigebrandt (1990), was presented in Stigebrandt & Aure (1990) and is further discussed in Aure et al. (1997).

The exchange of basin water (located beneath sill level) is controlled by the rate of vertical mixing in the basin and occurs when the density of coastal water at sill depth is high enough to replace resident basin water. In most fjord basins, the vertical mixing is driven essentially by energy transferred at sills from barotropic tides to baroclinic waves and later on to turbulence. In FjordEnv the mean residence time of basin water is estimated as described in Aure & Stigebrandt (1989a, 1990). In fjords with quite weak tidal currents across the sills, the wind may contribute significantly to mixing in the basin water. Wind-induced deepwater mixing is included in a pragmatic way in FjordEnv but it is not yet clear how this should be parameterised in a general way.

In the Møre project it was found that essentially all organic matter sinking down into the basin water of the fjords is imported from the offshore area with water entering the fjords just above sill level (Aure and Stigebrandt, 1989b). This was true especially in fjords where the residence time of water above the sill level is short and the sill is not too shallow. The contribution to the basin water by organic matter produced locally, i.e. in the surface water of the fjord, was thus usually small. On an annual basis, it seems that the vertical flux varies only slowly along a coast, see Stigebrandt et al. (1996) for a summary of the vertical flux of organic matter along the western coast of Scandinavia. If fish farms are placed in fjords in water deeper than the sill depth, surplus fish feed and faeces will be deposited in the basin water. FjordEnv computes the

increased rate of oxygen consumption and the decreased oxygen minimum in the basin water due to fish farming.

FjordEnv uses the fact that the external (offshore) forcing of water exchange and the vertical flux of organic matter into sill basins vary only slowly along a coast. Regional data on offshore forcing are stored in a database connected to the program making it easy to apply the model to regions where the offshore forcing already has been established.

In the following sections elements necessary to include in inshore water quality models are described. Section 2 gives a short description of oceanographic conditions and processes in coastal and inshore waters and some basic definitions. Section 3 discusses local forcing of importance to water quality. Section 4 describes different modes of water exchange. The following two sections, 5 and 6, deal with water quality in the surface layer and in the basin water, respectively. In several of the sections there are specific comments on implementations in the FjordEnv model and Section 7 gives a summary of data needed to run that model. Finally in Section 8 some concluding remarks are given together with suggestions to possible improvements of the model. The symbols used in this report are listed in an Appendix to the report.



## 2. Basic conditions and processes in coastal and inshore waters

Some key elements of the hydrography and dynamics of fjords are illustrated in Fig. 2.1. The surface water may have reduced salinity due to freshwater supply. It is kept locally well mixed by the wind and the thickness is typically of the order of a few meters. The thickness of the intermediary layer, reaching down to the sill level, depends strongly on the sill depth. This layer may be thin and even missing in fjords with shallow sills. Surface and intermediary waters have through the mouth free connection with the coastal area. Basin water, the densest water in the fjord, is trapped behind the sill. It is vertically stratified but the density varies less than in the layers above. A typical vertical distribution of density,  $\rho(z)$ , in a strongly stratified fjord is shown in Fig. 2.1. The area (water) outside the fjord is here denoted coastal area (water).

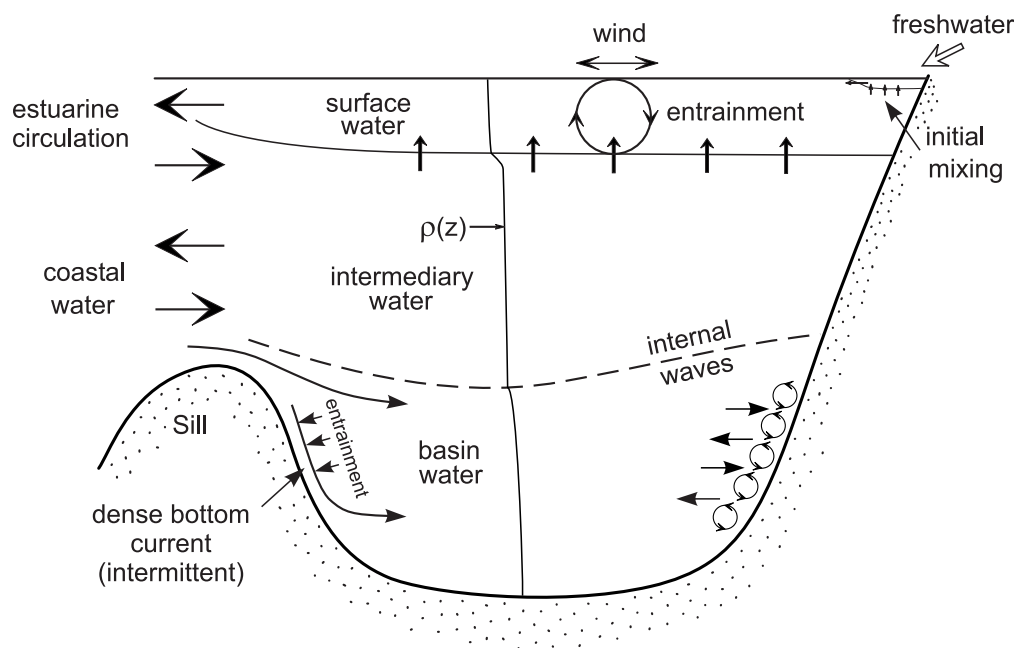


Fig. 2.1. Basic features of fjord hydrography and circulation

In most fjords, temporal variations of surface elevation and the density of the coastal water are crucial for the water exchange, both above and below the sill level. Surface and intermediary waters in short fjords with relatively wide mouths, may be exchanged in short time (days). The vertical stratification in the intermediary layer in such a fjord is usually quite similar to the stratification outside the fjord although there is some phase lag, with the coastal stratification leading before that in the fjord. Short residence time for water above sill level means that the pelagic ecology may change rapidly due to advection. Quite dense coastal water, occasionally appearing above sill level, intrudes the fjord and sinks down along the seabed. It then forms a turbulent dense bottom current that entrains ambient water, decreasing the density and increasing the volume flow of the current (Fig. 2.1). The intruding water replaces residing basin water. During so-called stagnation periods, when the density of the coastal water above sill level is less than the density of the basin water, a pycnocline develops in the fjord at or just below sill depth. During extended stagnation periods, oxygen deficit and even anoxia may develop in the basin water beginning at the greatest depths. The density of basin water decreases slowly due to

turbulent mixing, transporting less dense water from above into this layer. Tides usually provide most energy to deepwater turbulence in fjords. Baroclinic wave drag, acting on barotropic tidal flow across sills separating stratified basins, is the process controlling this energy transfer.

The topography of an inshore area may have great importance for the exchange and residence time of water in different depth strata. For the computations it is necessary to know the hypsographic function (area vs. depth) and the width of the mouth vs. depth. The sill depth  $H_t$  is the deepest connection between inshore and offshore areas. The horizontal surface area of the inshore area is  $A_f$  at the sea surface and  $A_t$  at sill depth. The volume of the inshore area is  $V$ . This may be partitioned into the volume above ( $V_t$ ) and below the sill depth ( $V_b$ ). The vertical cross-sectional area of the mouth is  $A_m$ . In some cases, islands split the mouth in several straits.

## 2.1 Light, sight depth and plankton concentration

The vertical penetration of light, of intensity  $I$  ( $\text{Wm}^{-2}$ ), may be described by the following equation

$$I = I_0 e^{-k_d z} \quad (2.1)$$

Here  $I_0$  is the intensity of light at the sea surface,  $k_d$  ( $\text{m}^{-1}$ ) is the vertical attenuation coefficient and  $z$  the depth. The attenuation coefficient gets contributions from absorption by the water itself and by dissolved and particulate matter. For pure seawater  $k_d \approx 0.04$ . It increases with the concentration of organic matter in the water. For example, at the coast of Skagerrak the water contains a lot of humic substances from the Baltic and typical winter values of  $k_d$  are about 0.10. During intense algae blooms in the same area  $k_d$  may increase to 0.3 and even higher.

It is often assumed that photosynthesis occurs in a layer at the sea surface of thickness  $D_f$  (m), limited downwards by the light intensity  $I$  being about 1% of  $I_0$ . This layer is called the euphotic zone or the production layer. From Eq. (2.1) above one may compute the depth of the production layer

$$D_f = -\frac{\ln(I / I_0)}{k_d} \quad (2.2)$$

With  $I/I_0=0.01$  and  $k_d=0.05$  (i.e. almost pure seawater) the thickness of the production layer becomes  $D_f=92$  m. For water with  $k_d=0.3$  the thickness of the production layer becomes  $D_f=15$  m.

The sight depth  $D_s$  is measured by visible disappearance of a white circular disc with the diameter 25 cm (a so-called Secchi disc) that is immersed into the water. The relationship between  $D_s$ , often denoted the Secchi depth, and  $k_d$  may be written

$$D_s = \frac{C_s}{k_d} \quad (2.3)$$

The magnitude of the empirical constant  $C_s$  will depend on properties of the pelagic ecosystem. Based on measurements in the Norwegian and Barent Seas (Aas, 1980), Stigebrandt and Aure (1989) estimated  $C_s=1.54$ . From Eq. (2.3) follows that one may estimate a layer mean  $k_d$  from Secchi depth measurements. With the numerical data given above one finds that the production layer is about 3 times greater than the Secchi depth.

The vertical attenuation coefficient  $k_d$  may be partitioned into two contributions – one from phytoplankton ( $k_p$ ) and one from the water itself plus dissolved matter and suspended mineral particles ( $k_b$ ), thus

$$k_d = k_b + k_p \quad (2.4)$$

Published data (e.g. Kirk, 1983) suggest that one may expect an approximately linear relationship between  $k_p$  and the concentration of phytoplankton measured as, for instance, particular organic phosphorus POP. The constant of proportionality may vary between different species of phytoplankton. The value 0.4 is used in FjordEnv so that

$$k_p = 0.4 \cdot POP \quad (2.5)$$

Here POP is expressed in  $\text{mmolPm}^{-3}$ . Eqs. (2.3), (2.4) and (2.5) give the following relationship between Secchi depth and the concentration of phytoplankton

$$D_s = \frac{C_s}{k_b + 0.4 \cdot POP} \quad (2.6)$$

This formula is used in FjordEnv to compute changes in Secchi depth due to changes in the supplies of nutrients to the surface layer.

Organic matter is assumed to be composed according to the so-called Redfield ratios (by moles)

$$C:N:P = 106:16:1 \quad (2.7)$$

The NP ratio is used to compute change of the production of POM due to change of nutrient supplies. The change of production is assumed controlled by the overall-limiting nutrient (i.e. at the system's level). In the computations of oxygen consumption, it is assumed that  $\mu$  grams of oxygen are needed for a complete oxidation of organic matter containing one gram of carbon ( $\mu \approx 3.5$  g Oxygen/1 g Carbon).

## 2.2 Vertical flux of organic matter and oxygen consumption in deeper layers

Studies of the oxygen consumption in many sill basins during stagnant conditions have shown that the flux of particulate organic matter (POM) into the basin water of fjords decreases with the sill depth, c.f. Fig. 2.2, and may be described by the following equation (Aure and Stigebrandt, 1990).

$$F_c = F_{co} e^{-z/L} \quad (2.8)$$

Here  $F_C$  is the flux of carbon contained in the POM. The value of  $F_C$  close to the sea surface,  $F_{C0}$ , varies slowly along the coasts but systematic investigations have so far only been published for the Norwegian coast. The length scale  $L_m$  for pelagic remineralisation should depend on properties of the pelagic ecosystem. Aure and Stigebrandt (1990b) estimated  $L_m$  to 50 m for the West Coast of Norway. The vertical flux of organic matter into a sill basin may be estimated from Eq. (2.8) with  $z=H_t$  where  $H_t$  is the sill depth.

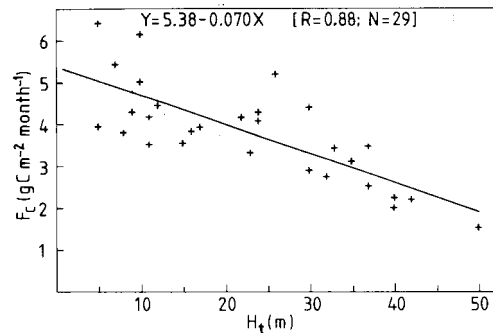


Fig. 2.2 The monthly vertical flux of POM as function of sill depth  $H_t$ . Data were obtained from fjords in Møre and Romsdal (from Aure and Stigebrandt, 1989b).

Assuming that POM has a mean (effective) sinking velocity  $w$ , one may transform a description in space to a description in time. This is done by the transformation  $t=w/z$ . The exponent in Eq. (2.8) may then be written  $-z/L_m=-t/T_m$  where  $T_m=L_m/w$  is the time scale for mineralisation and  $\ln(2)T_m\approx 0.7T_m$  is the half-life time of POM. If  $L_m=50$  m and  $w=1.5$  m/day, as found by Aure and Stigebrandt (1989a, 1990), one obtains  $T_m=1$  month. This means that about 3% ( $=1/T_m$ ) of available POM are consumed each day and the half-life time for pelagic POM is about 3 weeks. These figures are important for the understanding of how a cloud of POM evolves in space and time.

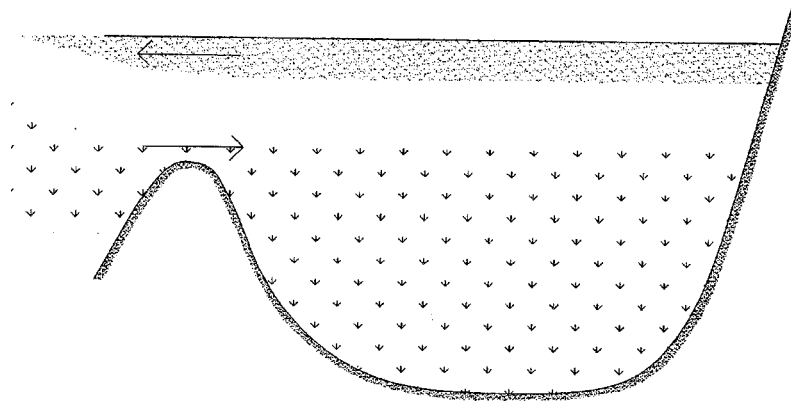


Fig. 2.3 POM produced in the surface layer is exported from fjords with short residence time for water above sill depth. The flux of POM into the sill basin then completely relies on import and determined by the offshore concentration of POM at sill level (From Stigebrandt, 1992).

Aure and Stigebrandt (1989a,b) concluded that Eq. (2.8) implies that local outlets of nutrients to fjords in general do not contribute to the flux of POM into the basin water. At the first glance, this may seem astonishing. The explanation is that due to low sinking speed POM produced by local outlets of nutrients to the fjord will be far outside the fjord before having reached the sill level, cf. Fig. 2.3. This will be true in fjords where the residence time for water above sill level is less than the time it takes for POM produced by local outlets to sink to sill level. If this is not the case, local outlets may indeed contribute to the flux of POM into the basin water. There are also other cases where Eq. (2.8) does not quite well describe the flux of POM into the basin water as discussed in Section 6.1 later in this paper where also a more generally applicable formula is presented.

### 3. Supplies of plant nutrients, organic matter and freshwater

The contents of N and P in fish feed depend on the composition of the feed. For the computations it is assumed that so-called standard feed is used containing 45% protein, 30% fat and 7% carbohydrates by weight, the rest is ashes and water. N and P are essentially contained in the protein. Most of this is incorporated in growing fish but some is excreted in soluble forms and by faeces. The values in Table 3.1 were obtained from the fish model in Stigebrandt (1999b).

The supply of P and N from other sources should be given (tons/year). It is assumed that the plant nutrients emitted by the fish to the water in the cages are supplied to the surface layer.

The supply of freshwater is important for the estuarine circulation and for computations of changes of the Secchi depth. The supply relevant for the production season should be given.

Table 3.1 Oxygen demand by the fish and waste products from 1 ton of fish production. The feed is composed of 45% protein, 30% fat and 7% carbohydrates. It is assumed that the excess feed is 30% of the ingested feed (from Stigebrandt, 1999b).

Surface layer (cages)	Oxygen demand (kg)	520
	Excretion, ammonium (kg)	28
	Excretion, phosphorus (kg)	4.5
Benthic layer	Faeces, carbon (kg)	50
	Faeces, nitrogen (kg)	16
	Faeces, phosphorus (kg)	2.5
	Excess feed, carbon (kg)	135
	Excess feed, nitrogen (kg)	19
	Excess feed, phosphorus (kg)	3.2

#### 3.1 Specific comments concerning the FjordEnv implementation

In FjordEnv annual fish production over deep and shallow areas (i.e. deeper or shallower than the sill depth) and excess feed (in percent of the feed consumed by the fish) must be specified. For the computations FjordEnv assumes that standard feed is used.

## 4. Modes of water exchange

### 4.1 Basic physical processes

Understanding the circulation in fjords and other inshore areas requires knowledge of some basic physical processes like diapycnal mixing and a variety of strait flows, regulating the flow through the mouth. Natural modes of oscillation (seiches) as well as sill-induced processes like baroclinic wave drag (generating internal waves in fjords), tidal jets and internal hydraulic jumps may constitute part of the fjordic response to time-dependent forcing. The most important physical processes responsible for horizontal and vertical transports in inshore areas are briefly discussed in this section.

#### 4.1.1 Strait flows

Flows through fjord mouths are mainly driven by barotropic and baroclinic longitudinal pressure gradients. The barotropic pressure gradient, constant from sea surface to seabed, is due to differing sea levels in the fjord and the coastal area. Baroclinic pressure gradients arise when the vertical density distributions in the fjord and in the coastal area differ. The baroclinic pressure gradient varies with depth. Different types of flow resistance and mechanisms of hydraulic control may modify flow through a mouth. Barotropic forcing usually dominates in shallow fjord mouths while baroclinic forcing may dominate in deeper mouths.

Three main mechanisms cause resistance to barotropic flow in straits. Firstly, friction against the seabed. Secondly, large-scale form drag, due to large-scale longitudinal variations of the vertical cross-sectional area of the strait causing contraction followed by expansion of the flow. Thirdly, baroclinic wave drag. This is due to generation of baroclinic (internal) waves in the adjacent stratified basins.

Barotropic flow  $Q$  through a straight, rectangular, narrow and shallow strait of width  $B$ , depth  $D$  and length  $L$ , connecting a wide fjord and the wide coastal area may be computed from the following equation

$$Q^2 = \frac{2g\Delta\eta B^2 D^2}{1 + 2C_D(L/D)} \quad (4.1)$$

The sign of  $Q$  equals that of  $\Delta\eta = h_o - h_i$ ,  $h_o$  ( $h_i$ ) is the sea level in the coastal area (fjord). This equation includes resistance due to both large-scale topographic drag and bottom friction (drag coefficient  $C_D$ ) but not due to baroclinic wave drag, see Stigebrandt (1999c).

To get first estimates of baroclinic flows and processes, it is often quite relevant to approximate a continuous stratification by a two-layer stratification, with two homogeneous layers of specified thickness and density difference  $\Delta\rho$  on top of each other. The buoyancy of the less dense water is  $g' = g\Delta\rho/\rho_0$ , where  $g$  is the acceleration of gravity and  $\rho_0$  a reference density.

In stratified waters, fluctuating barotropic flows (e.g. tides) over sills are subject to baroclinic wave drag. This is important in fjords because much of the power transferred to baroclinic

motions apparently ends up in deepwater turbulence. Assuming a two-layer approximation of the fjord stratification, with the pycnocline at sill depth, the barotropic to baroclinic energy transfer  $E_j$  from the  $j$ :th tidal component is (Stigebrandt, 1976)

$$E_j = \frac{\rho_0}{2} \omega_j^2 a_j^2 \frac{A_f^2}{A_m} \frac{H_b}{H_b + H_t} c_i \quad (4.2)$$

Here  $\omega_j$  is frequency and  $a_j$  amplitude of the tidal component.  $A_f$  is the horizontal surface area of the fjord,  $A_m$  the vertical cross-sectional area of the mouth,  $H_t$  ( $H_b$ ) sill depth (mean depth of the basin water) and  $c_i = \sqrt{g' \frac{H_t H_b}{H_t + H_b}}$  the speed of long internal waves in the fjord. Baroclinic wave drag occurs if the speed of the barotropic flow in the mouth is less than  $c_i$ . A fjord in this range is denoted 'wave fjord'. If the barotropic speed is higher, a tidal jet develops on the lee side of the sill ('jet fjord').

A tidal jet may be accompanied by a number of flow phenomena like internal hydraulic jumps and associated internal waves of super-tidal frequencies. The mean energy loss of the tidal component  $j$  to a jet in the fjord is (Stigebrandt and Aure, 1989)

$$E_{jet,j} = 0.42 \rho u_{s0,j}^3 \frac{A_m}{4} \quad (4.3)$$

Here  $u_{s0,j}$  is defined by

$$u_{s0,j} = \frac{A_f}{A_m} a_j \omega_j \quad (4.4)$$

Baroclinic flows in straits may be influenced by stationary internal waves imposing a baroclinic hydraulic control. For a two-layer approximation of the stratification in the mouth the flow is hydraulically controlled if the following condition, formulated by Stommel and Farmer (1953) is fulfilled

$$\frac{u_{1m}^2}{g' H_{1m}} + \frac{u_{2m}^2}{g' H_{2m}} = 1 \quad (4.5)$$

Here  $u_{1m}$  ( $u_{2m}$ ) and  $H_{1m}$  ( $H_{2m}$ ) are speed and thickness, respectively, of the upper (lower) layer in the mouth. Equation (4.5) may serve as a dynamic boundary condition for baroclinic fjord circulation. An example is provided in the model for estuarine circulation in Section 4.2 below. Experiments show that superposed barotropic currents just modulate the flow (Stigebrandt, 1977). However, if the barotropic speed is greater than the speed of internal waves in the mouth these are swept away and a baroclinic hydraulic control cannot be established. In wide fjords, the rotation of the earth may limit the width of baroclinic currents to the order of the internal Rossby radius  $\sqrt{g' H_1} / f$ . Here  $H_1$  is the thickness of the upper layer in the fjord and  $f$  the Coriolis



parameter. Steady outflow from the surface layer is then essentially geostrophically balanced and the transport is

$$Q_{bc} = g' H_1^2 / 2f$$

This expression has been used as boundary condition for the outflow of surface water from e.g. Kattegat (Stigebrandt, 1983; Gustafsson, 2000) and the Arctic Ocean (polar surface water) through Fram Strait (Stigebrandt, 1981b; Björk, 1989).

Stratified bays, fjords and other inshore waters respond to time-dependent density changes in the adjacent coastal water in such a way that the vertical stratification tends to that in the coastal water. The obvious reason for this is that density changes in the coastal water induce horizontal baroclinic pressure gradients and by that currents between inshore and offshore waters. In fjords the resulting water exchange has been termed intermediary water exchange since it is most easily observed in the intermediary water layers, situated between the surface top layer and the fjord sill. Published examples of externally (remotely) forced vertical isopycnal displacements in fjords are given in e.g. Aure et al. (1997). The intermediary circulation is often an order of magnitude greater than circulation induced by freshwater supply (estuarine circulation) and surface tides, see Stigebrandt and Aure (1990). The physics of intermediary circulation is discussed in Stigebrandt (1990) where also an approximate numerical model is presented. A deeper analysis of remotely forced baroclinic flows in straits is presented in Engqvist (1996).

#### 4.1.2 Diapycnal mixing processes

Diapycnal (vertical) mixing processes may modify the water masses in fjords. In the surface layer, the wind creates turbulence that homogenises the surface layer vertically and entrains seawater from below. The rate of entrainment may be described by a vertical velocity,  $w_e$ , defined by the following expression

$$w_e = \frac{m_0 u_*^3}{g' H_1} \quad (4.6)$$

Here  $u_*$  is the friction velocity in the surface layer, linearly related to the wind speed,  $m_0$  ( $\approx 0.8$ ) is essentially an efficiency factor, well known from seasonal pycnocline models, see e.g. Stigebrandt (1985).  $H_1$  is the thickness of the surface layer in the fjord and  $g'$  the buoyancy of surface water relative to the underlying water.

Due to their sporadic and ephemeral character, there are no direct observations of dense bottom currents in fjords, e.g. Edwards and Edelsten (1980) and Liungman et al. (2000). However, from both observations and modelling of dense bottom currents in the Baltic it appears that the entrainment velocity may be described by Eq. (4.6). For this application  $u_*$  is proportional to the speed of the bottom current,  $H_1$  is the current thickness and  $g'$  the buoyancy of ambient water relative to the water in the dense bottom current, see Stigebrandt (2001) for a discussion of the conditions in the Baltic. There is, however, no explicit modelling of dense bottom currents in FjordEnv.

Observational evidence strongly supports the idea that diapycnal mixing in the basin water of most fjords is driven essentially by tidal energy, released by baroclinic wave drag at sills. It should be noted that in fjords with weak tides, deepwater turbulence may get quite significant contributions of energy through dissipation of surface seiches (Parsmar and Stigebrandt, 1997) and internal seiches (Arneborg and Liljebladh, 2001). Also in these cases it seems that baroclinic wave drag at sills play a crucial role for the transfer of energy to turbulence in the basin water. The details of the energy cascade, from baroclinic wave drag to small-scale turbulence, are still not properly understood. Fig. 2.1 leaves the impression that energy transfer to small-scale turbulence and diapycnal mixing takes place in the inner reaches of a fjord. Here internal waves, for instance generated by baroclinic wave drag at the sill, are supposed to break against sloping bottoms. The temporal and spatial distributions of turbulent mixing in basin waters of fjords have not yet been mapped. However, a tracer experiment in the Oslo Fjord suggests that mixing essentially occurs along the rim of the basin, possibly close to the sill where the main part of the barotropic to baroclinic energy transfer takes place, see e.g. Stigebrandt (1999a) for a review. Observations supporting this view have recently been reported from lakes (Gloor et al., 2000).

The mean rate of work against the buoyancy forces in a column of the basin water,  $W$ , may be computed from the following expression, see Stigebrandt and Aure (1989)

$$W = W_0 + \frac{Rf \sum_{j=1}^n E_j}{A_t} \quad (4.7)$$

Here  $W_0$  is the contribution due to non-tidal energy supply,  $n$  the number of tidal components,  $E_j$  may be obtained from either Eq. (4.2) (for wave fjords) or Eq. (4.3) (jet fjords) as further discussed in section 4.5 below.  $A_t$  is the horizontal surface area of the fjord at sill level and  $Rf$ , the flux Richardson number, the efficiency of turbulence with respect to diapycnal mixing. Estimates from numerous fjords show that  $Rf$  equals about 0.06. Experimental evidence presented by Stigebrandt and Aure (1989) shows that in jet fjords, most of the released energy dissipates above sill level and only a small fraction contributes to mixing in the deepwater. In this case,  $Rf$  in Eq. (4.7) should be taken equal to 0.01.

## 4.2 Estuarine circulation

The surface layer in a fjord receives freshwater, mostly by run-off from land, and seawater from below by wind-driven entrainment. It loses water by outflow through the fjord mouth. A model of the surface layer should account for these fluxes. If barotropic tidal velocities are relatively small, the baroclinic flow in the mouth attains the phase speed of internal waves (critical flow) (Stommel and Farmer, 1953). For the model, one needs to apply the condition of critical flow in the mouth and use an appropriate parameterisation of the entrainment of seawater into the surface water. One also has to compute the thinning of the surface layer from the fjord interior to the critical section due to acceleration.

For stationary conditions, conservation of volume gives

$$Q_1 = Q_f + Q_2 \quad (4.8)$$

Here  $Q_{1(2)}$  is outflow (inflow) through the mouth and  $Q_f$  is the freshwater supply.

Conservation of salt gives

$$Q_1 S_1 = Q_2 S_2 \quad (4.9)$$

Here  $S_{1(2)}$  is the salinity of the surface layer (seawater).

Fjords are often so strongly salt-stratified that the contribution by vertically varying heat content to the stratification may be disregarded. The following equation of state for brackish water may then be used

$$\rho = \rho_f (1 + \beta S) \quad (4.10)$$

Here  $\rho_f$  is the density of freshwater,  $\beta$  the salt contraction coefficient ( $\approx 0.0008 \text{ \%}^{-1}$ ) and  $S$  salinity ( $\text{‰}$ ).

If one assumes that the lower layer is thick and moves with low velocity, Eq. (4.5) can be simplified. With this assumption, one may obtain the following analytical solution for the steady state thickness and salinity of the surface layer of the fjord (Stigebrandt, 1975, 1981a).

$$H_1 = \frac{N}{2Q_f M} + \varphi \left( \frac{Q_f^2}{MB_m^2} \right)^{\frac{1}{3}} \quad (4.11)$$

$$S_1 = \frac{S_2 N}{N + 2\varphi \left[ Q_f^5 \left( \frac{M}{B_m} \right)^2 \right]^{\frac{1}{3}}} \quad (4.12)$$

Here  $A_f$  is the surface area of the fjord,  $B_m$  the width of the mouth,  $W$  the mean wind speed,  $M=g\beta S_2$  and  $N=C_e W^3 A_f$  where  $C_e=2.5 \cdot 10^{-9}$  is an empirical constant containing, among others, the drag coefficient for air flow over the sea surface. The thickness of the surface layer decreases during acceleration towards the critical section in the mouth.  $\varphi$  is the ratio between the thickness' of the surface layer in the fjord and in the mouth, respectively. Theoretically, the value of  $\varphi$  is expected to be in the range 1.5-1.75. Observations in fjords give  $\varphi$ -values in the range 1.5-3, see Stigebrandt and Molv ar (1997).

The first term on the right hand side of Eq. (4.11) is the Monin-Obukhov length determined by the buoyancy flux ( $=g\beta S_2 Q_f/A_f$ ) and wind mixing ( $\propto W^3$ ). The second term is the freshwater thickness determined by the hydraulic control. The salinity and thickness of the surface layer in Nordfjord as computed by this model are shown in Fig. 4.1.

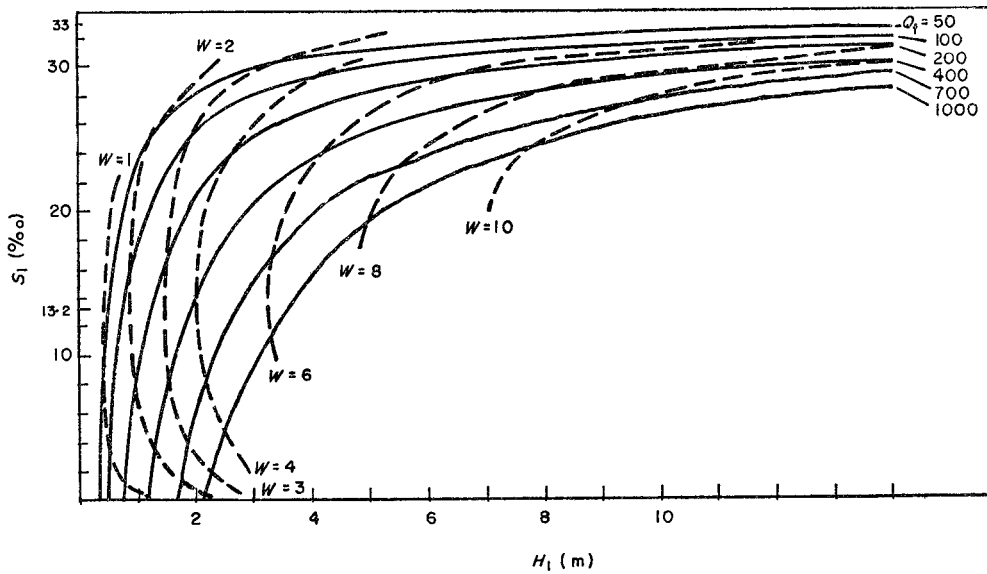


Fig. 4.1 The estuarine circulation in Nordfjord ( $A_f= 300 \text{ km}^2$ ,  $B_m= 1200 \text{ m}$ ,  $S_2=33$ ,  $\varphi=1.5$ ) for various values of the wind speed  $W$  and freshwater supply  $Q_f$  as computed from Eqs. (4.11) and (4.12). Note that for constant wind speeds the surface layer attains its minimum thickness when  $S_1=0.4 \cdot S_2=13.2$ . From Stigebrandt (1981a).

Eqs. (4.8) and (4.9), often called Knudsen's relationships, can be combined to get the following expressions for the components of the estuarine circulation

$$Q_1 = \frac{Q_f S_2}{S_2 - S_1} \quad (4.13)$$

$$Q_2 = \frac{Q_f S_1}{S_2 - S_1} \quad (4.14)$$

The latter equation gives the estuarine circulation ( $Q_e=Q_2$ ) when  $S_1$  has been computed from Eq. (4.12).

For a shallow estuarine circulation, like the one treated above, the residence time for water in the surface layer  $T_f$  is

$$T_f = \varphi A_f \left( \frac{1}{MB_m^2} \right)^{\frac{1}{3}} Q_f^{-1/3} \quad (4.15)$$

It should be noted that  $T_f$  decreases slowly with increasing freshwater supply and is independent of the rate of wind mixing.

For small freshwater supplies and strong winds, the salinity difference  $S_2 - S_1$  might become quite small and the estuarine circulation very large. The assumption of low velocities in the lower layer is then no longer fulfilled and Eqs. (4.11) and (4.12) are no longer valid. The full version of Eq. (4.5) has to be used in this case for which no simple analytical solutions for  $H_1$  and  $S_1$  are known.

#### 4.2.1 Specific comments concerning the FjordEnv implementation

The estuarine circulation and the salinity and depth of the surface layer may be computed separately in FjordEnv for various combinations of  $Q_f$ ,  $W$ ,  $S_2$  and  $\varphi$ .

In fjords with strong fluctuating barotropic currents in the mouth, the baroclinic control of the water exchange may be invalidated for long periods as described in Section 4.1 above. The water exchange is then instead controlled by the barotropic flow. The estuarine circulation is then strongly dependent on the characteristics of the barotropic flow through the mouth. This is for instance the case in the mouth of the Baltic Sea, see e.g. Stigebrandt (2001). FjordEnv handles a simple variant of this case.

### 4.3 Intermediary circulation

Despite its often-dominating contribution to water exchange between inshore and offshore waters, the intermediary circulation has remained astonishingly anonymous and in many studies of inshore waters even completely overlooked. One reason for this is that until recently simple models to quantify intermediary circulation under realistic conditions have not been available. Klinck et al. (1981) used a two-layer model to compute fjordic response to a slowly varying pycnocline oscillation in the coastal water. A numerical model that computes water exchange between fjords and coastal waters with high temporal and vertical resolution was presented in Stigebrandt (1990). Stigebrandt and Aure (1990) applied this model to a wide spectrum of fjords with the aim to develop an analytical formula for intermediary water exchange  $Q_i$ . The following formula was fitted to the model results

$$Q_i = \gamma \left( g B_m H_t A_f \frac{\Delta M}{\rho} \right)^{\frac{1}{2}} \quad (4.16)$$

Here  $B_m$  is the width of the mouth,  $H_t$  the sill depth,  $A_f$  the surface area of the fjord. The forcing of the intermediary circulation is given by the standard deviation  $\Delta M$  of the weight  $M$  ( $\text{kg m}^{-2}$ ) of a water column from the mean sea surface down to sill depth.  $\Delta M$  is computed by vertical integration of the standard deviation  $\sigma_\rho(z)$  of observations of the density  $\rho(z)$  where  $z$  is the depth co-ordinate. The value of the empirical, dimensionless, constant  $\gamma$  was estimated to  $17 \cdot 10^{-4}$ . The model for intermediary circulation in Stigebrandt (1990) has been used by a number of authors, see Aure et al. (1997) where also a field program to test Eq. (4.16) is presented.

The forcing of the intermediary circulation in Eq. (4.16) was chosen to be represented by  $\Delta M$  because this gives an integrated measure of the baroclinic variability down to sill depth. Furthermore,  $\Delta M$  may be computed using statistics of scattered historical hydrographic measurements. Thus, historical hydrographic measurements are of great value because they make possible estimates of the mean intermediary circulation of inshore water areas along coastal stretches even if no representative time series of the vertical stratification are available. Stigebrandt and Aure (1990) tabulated  $\Delta M$  for various depths for the locations along the Norwegian coast where Institute of Marine Research, Bergen regularly takes hydrographic measurements. Stigebrandt (2001) tabulated  $\Delta M$  for different parts of the Baltic see Table 4.1.

From an investigation of the spatial variation of the variability of the density field  $\sigma_\rho^2(x,y,z)$  in Skagerrak, it was found that  $\sigma_\rho^2$  decreases monotonously with depth (Gustafsson and Stigebrandt, 1996). In the surface layers (0-20 m) the variability is maximum at the Kattegat border and, going cyclonically along the Skagerrak coast, it decreases with the distance from Kattegat (Fig. 4.2). At greater depths the variability has maxima outside the Norwegian and Danish coasts, primarily due to events with deep-reaching downwelling (Fig. 4.3). At all coastal stretches, the variability decreases generally in the seaward direction. For a more detailed description of the variability of the density field in Skagerrak the reader is referred to Gustafsson and Stigebrandt (1996).

Eq. (4.16) is valid provided the ratio between the surface area of the fjord and the vertical cross-sectional area of the mouth is less than about  $10^4$ . If the ratio is greater, the intermediary water exchange should be computed from the following formula

$$Q_i = \frac{B_m H_t}{6} \sqrt{\frac{2g\Delta M}{\rho}} \quad (4.17)$$

This formula, derived in Stigebrandt and Aure (1990), implies that the flow is hydraulically controlled in the mouth in the sense of Eq. (4.5). Eqs. (4.16) and (4.17) should also be valid for other kinds of inshore areas.

Table 4.1 shows that  $\Delta M$  varies by a factor of about 6 along the coasts of western Scandinavia and the Baltic Sea with maximum in Kattegat where low salinity water from the Baltic meets saline water from Skagerrak. For topographically similar inshore areas covered by Table 4.1 the intermediary circulation may vary by about a factor of about 2.5 ( $\approx\sqrt{6}$ ).

Table 4.1 The forcing of intermediary water exchange,  $\Delta M$  ( $\text{kg m}^{-2}$ ), for some depths along the coasts of western Scandinavia and the Baltic. From Stigebrandt and Aure (1990) and Stigebrandt (2001).

Depth (m)	Coast of Barents Sea	Lofoten area	Outer Skagerrak	Northern Kattegat	Baltic proper	Bothnian Sea
10	4	8	20	32	6	5
20	7	15	35	53	10	7
30	10	20	44	67	14	9
40	13	26	51	77	18	11
50	16	30	56	87	23	13

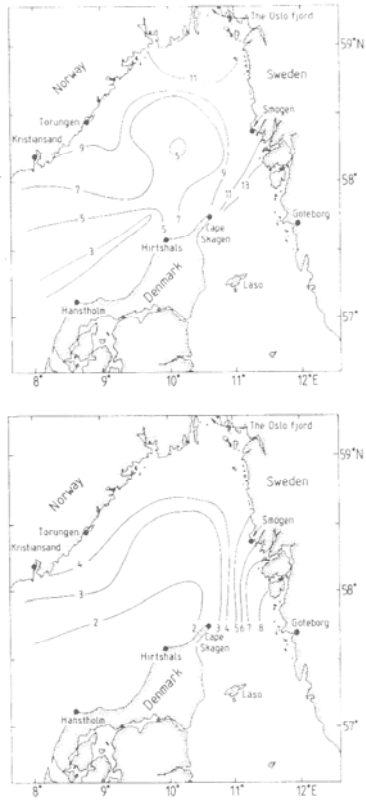


Fig. 4.2 Horizontal variation of the variability ( $\sigma_{\rho}^2(x,y)$ ) of the density field in Skagerrak at the sea surface (upper) and the depth 10 m (lower). From Gustafsson and Stigebrandt (1996).



## 4.4 Tidal pumping

The offshore tide acts like a pump that during half the tidal cycle sucks water out of an inshore area and during the next half cycle forces about the same amount of water into the area. The mean volume transport  $Q_t$  into and out of an inshore area during a tidal period of length  $T$  is

$$Q_t = 2a_o cc \frac{A_f}{T} \quad (4.18)$$

Here  $a_o$  is the amplitude in the coastal area and  $cc$  is the so-called choking coefficient defined by  $cc = a_i / a_o$  where  $a_i$  is the mean tidal amplitude in the inshore area. The amplitude  $a_i$  may be substantially reduced, as compared to  $a_o$ , due to friction and/or topographical resistance in the mouth as described in section 4.1 above.

Some of the water flowing out of the inshore area during ebb may return during flood and vice versa. The effective water exchange forced by semidiurnal tides should therefore in many cases be less than expected from Eq. (4.18). One should therefore multiply the right hand side of Eq. (4.18) by an efficiency factor  $\varepsilon$  ( $0 < \varepsilon < 1$ ). There is no simple method to compute the magnitude of  $\varepsilon$ . It is possible that superposed baroclinic modes of water exchange will increase the efficiency factor.

In the Baltic, the tide is virtually absent and tidal pumping should be negligible in Baltic inshore areas although pumping due to meteorologically forced sea level variations may be important in some areas. The tidal amplitude increases northwards along the coast of western Scandinavia, from a few centimetres in Kattegat to about 1.4 metres in Kirkenes at the coast of Barents Sea.

### 4.4.1 Specific comments concerning the FjordEnv implementation

In FjordEnv  $\varepsilon$  is taken equal to 0.5 which usually should give a conservative estimate of the tidally forced water exchange. For the tidal amplitude  $a_o$  FjordEnv uses the sum of the amplitudes of  $M_2$  and  $S_2$  but other tidal and meteorological components are disregarded. This should also contribute to a conservative estimate.

## 4.5 Deepwater circulation

Tides play a major role for mixing in the basin water of fjords. Increased tidal velocities across sills usually give increased supply of mixing energy that leads to increased rates of mixing of less dense water into basins. An increased rate of density reduction leads to more frequent exchanges of basin water and by that improved oxygen conditions.

Oscillating tidal currents across a sill may generate internal tides and other internal waves in an adjacent stratified basin (wave fjord). This was described in section 4.1 where the basic physics for this section is covered. The energy transfer from barotropic tides at sills to internal tides is calculated according to Eq. (4.2). The energy in the internal tides is assumed to be transferred to turbulence in the basin water.

The total work  $W$  against the buoyancy forces per unit time and unit horizontal surface area in the basin water is given by Eq. (4.7), c.f. Fig. 4.3 and Section 4.5.1.

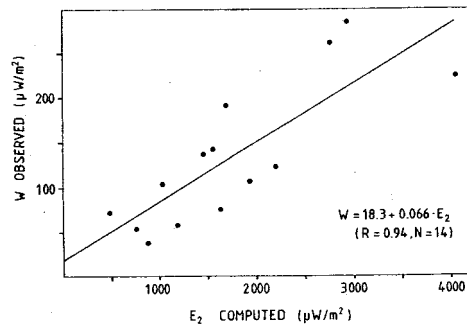


Fig. 4.3 The relationship between  $W$  and  $E/A_t$  for wave-fjords in Møre and Romsdal. From Stigebrandt and Aure (1989).

The background mixing  $W_0$  is thought to be due to the wind. The wind conditions vary from fjord to fjord. Stigebrandt and Aure (1989) found that the mean  $W_0$  for the investigated fjords in Møre and Romsdal ( $A_f$  typically  $10 \text{ km}^2$ ) in Norway is  $0.02 \text{ mW m}^{-2}$ , cf. Fig. 4.3. In the much larger Baltic Sea ( $A_f$  about  $10^5 \text{ km}^2$ ) where the wind is stronger than in small protected fjords  $W_0 \approx 0.10 \text{ mW m}^{-2}$ . From these two extremes it is assumed that  $W_0$  ( $\text{mW m}^{-2}$ ) increases with the horizontal surface area of the fjord  $A_f$  ( $\text{km}^2$ ) according to the following expression

$$W_0 = 10^{(0.2 \cdot \log A_f - 2)} \quad (4.19)$$

This equation is of course very uncertain and should be checked when data become available.

In Aure and Stigebrandt (1989a) a method was developed to estimate the time lapsed between two consecutive complete exchanges of basin water in fjords. It was demonstrated that the rate of density reduction  $d\rho/dt$  in basin water is proportional to  $W$ , and the following relationship was suggested

$$\frac{d\rho}{dt} = -\frac{C_W W}{gH_b^2} \quad (4.20)$$

The empirical constant  $C_W=2.0\pm 0.6$  and  $W$  is obtained from Eq. (4.7). The observational results are shown in Fig. 4.4.

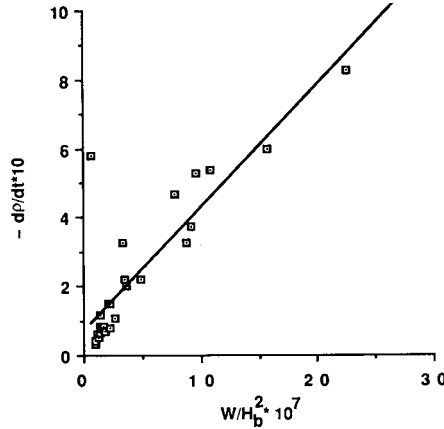


Fig. 4.4 The rate of density reduction in the basin water of fjords vs the dissipation rate (from Aure and Stigebrandt, 1990).

One may expect that the basin water is completely exchanged during the period  $T_e$  defined by

$$T_e = \frac{R_e}{\frac{d\rho}{dt}} = -\frac{R_e g H_b^2}{C_W} \quad (4.21)$$

Here  $R_e$  is the mean density reduction in the basin water needed to obtain a complete exchange of basin water. Empirical data from fjords in Møre and Romsdal show that one may expect basin water renewal when the mean density reduction since the last exchange is  $R_e \approx -4/3$  ( $\text{kg m}^{-3}$ ). It should be pointed out that the water higher up in the basin is exchanged more often and the effective  $R_e$ -value thus decreases upwards.

In Aure and Stigebrandt (1989a) it was argued that the value of  $R_e$  should be a function of the characteristics of the density fluctuations in the coastal water. For instance, if the density variability at sill depth in the coastal water is small, the  $R_e$ -value should also be small and vice versa. It may be assumed that the variability of the density in the coastal water may be characterised by the observed standard deviation. Data from Møre and Romsdal suggest that  $R_e = -1.5 \sigma_\rho(z=H_i)$ . For given depths, the standard deviation  $\sigma_\rho$  decreases northwards along the Norwegian coast.

For fjords with quite narrow mouths the absolute value of  $R_e$  is greater than  $1.5 \sigma_\rho$ . The reason for this is that the transport capacity of the mouth is relatively small due to the topographic restriction and the basin water cannot be exchanged completely in the limited time the density of the water outside the sill is dense enough. If the ratio between the volume of the basin water and

the vertical cross-sectional area of the sill is greater than about 70000 (m) the transport capacity of the mouth will influence the  $R_e$  –value in such a way that this increases (Aure and Stigebrandt, 1989a). Another measure of the transport capacity of a mouth is given by the time it takes to fill the basin with new water. If the mean rate of water exchange is  $Q$  the exchange requires the time  $T_{fill}$  to be complete.  $T_{fill}$  is defined by

$$T_{fill} = \frac{V_b}{Q} \quad (4.22)$$

The water exchange will be only partial if  $T_{fill}$  is longer than the time sufficiently dense water appears above sill level outside the mouth. This kind of fjords will probably have less dense deepwater in than adjacent fjords with more open mouths.

#### 4.5.1 Specific comments concerning the FjordEnv implementation

In FjordEnv the energy transfer to internal tides is estimated from the semidiurnal tide ( $M_2 + S_2$ ). The fraction  $1/\phi$  ( $0 < 1/\phi < 1$ ) of the total barotropic to baroclinic energy transfer comes from the semidiurnal tide. In FjordEnv it is assumed that  $1/\phi$  increases gradually, from the value 0.5 in Kattegat and Skagerrak to 0.95 in Barents Sea where. The energy transfer  $E_2$  to semi-diurnal internal tides (frequency  $\omega$ ) is calculated from the following variant of Eq. (4.2)

$$E_2 = \rho \omega^2 a_i^2 \frac{A_f^2}{2A_m} \frac{H_b}{H_b + H_t} c_i \quad (4.23)$$

Here  $a_i$  is the semidiurnal sea level amplitude in the fjord and  $c_i$  is the speed of internal waves in the fjord defined by

$$c_i = \sqrt{g \frac{\Delta\rho}{\rho_0} \frac{H_t H_b}{H_t + H_b}} \quad (4.24)$$

Here  $\Delta\rho = \rho_b - \rho_t$  and  $\rho_0$  is a reference density. In FjordEnv  $\Delta\rho$  is computed from  $\sigma_\rho$ .

In FjordEnv the mean energy transfer to a jet in the fjord by the semidiurnal tide is calculated from the following variant of Eq. (4.3)

$$E_{jet} = 0.42 \rho_0 u_{s0}^3 \frac{A_m}{4} \quad (4.25)$$

where

$$u_{s0} = \frac{A_f}{A_m} a_i \omega \quad (4.26)$$

In FjordEnv the total work against the buoyancy forces per unit time and unit horizontal surface area in the basin water is computed from the following variant of Eq. (4.7)

$$W = W_0 + \frac{Rf\phi E}{A_t} \quad (4.27)$$

Here  $W_0$  is the so-called background work performed by non-tidal forcing and  $Rf$  is the Richardson flux number, cf. Fig. 4.3.  $E$  is computed from Eq. (4.23) for wave fjords and from Eq. (4.25) for jet fjords. For the former  $Rf=0.06$  and for the latter  $Rf=0.01$ , see Section 4.1 above.

According to Eq. (4.21), the residence time for basin water should be inversely proportional to the rate of diapycnal mixing. This implies that there are possibilities to reduce the residence time by e.g. the introduction of artificial mixing in the basin water. One way of doing this is to change the topography of the mouth in such a way that  $W$  increases by e.g. road constructions, dredging etc. However, such manipulations must be checked so that the time  $T_{\text{fill}}$  does not increase so much that  $R_e$  increases. Since the width of the mouth is given in FjordEnv, this model may give first estimates of effects on the basin water by changes of the topography of the mouth.

## 5. Water quality in the surface layer – Secchi depth

Due to changes of the nutrient supply to the inshore surface layer, the Secchi depth will change from a known depth. The change in Secchi depth is computed as described below.

Nutrients coming from local sources, e.g. by local runoff and dissolved from fish farms, may directly be used by phytoplankton in the fjord. It is assumed that the nutrients are mixed into the surface layer. How the contribution from fish farming is estimated is described in Chapter 3. The nutrient supply from fish farming will vary with number and weight of fish, temperature and food composition as described in Stigebrandt (1999b).

Water exchange in the surface layer is assumed to be proportional to the total water exchange above sill depth. The constant of proportionality is assumed to be  $V_s/V_t$  where  $V_s$  is the volume of the surface layer and  $V_t$  is the volume of the fjord above sill level. In addition comes the net outflow caused by the freshwater supply and the estuarine circulation. If the latter two are assumed to take place in the surface layer, the water exchange  $Q_s$  in this layer is

$$Q_S = Q_f + Q_e + \frac{V_s}{V_t}(Q_i + Q_t) \quad (5.1)$$

The changed concentration of phosphorus in the surface layer,  $c_{Pf}$  ( $\text{mmol P m}^{-3}$ ), due to changed excretion by fish in fish farms and outlets from human activities is then

$$c_{Pf} = \frac{P_f}{Q_S} \quad (5.2)$$

The changed supply of phosphorus is  $P_f$  ( $\text{mmol s}^{-1}$ ). If it is assumed that all the phosphorus is used for plant production, the concentration of plankton (measured as P) increases by  $c_{Pf}$ . The Secchi depth then becomes

$$D_S = \frac{C_S}{\frac{C_S}{D_0} + 0.4 \cdot c_{Pf}} \quad (5.3)$$

Here  $D_0$  is the “normal” Secchi depth (i.e. the depth before the supply is changed by  $P_f$ ) and Eq. (2.5) has been used. Eq. (5.3) may of course also be used to estimate increases in Secchi depth for cases with decreasing supply of nutrients (negative values of  $P_f$ )

The computations above were done for P. One may equally well do the computations for N if this is considered to limit the production of organic matter.

The Secchi depth computed from Eq. (5.3) above may be too small (i.e. the environmental effect is overestimated) if phytoplankton do not use the nutrients before these are exported to offshore areas. This may possibly be the case in small fjords with rapid water exchange. The computations will also be uncertain if the surface layer in which the local production takes place is deeper than  $D_s$ .

If conditions in the offshore water change  $D_0$  will also change. This should be accounted for in MARE applications where changes of the state of the Baltic are considered.

### **5.1. Specific comments concerning the FjordEnv implementation**

In FjordEnv the thickness of the surface layer has been chosen to 5m. The reason for this is that net pens often are 5 m thick and FjordEnv originally was designed to cope with eutrophication effects of fish farming.

In FjordEnv computations are done for the nutrient that limits the production, c.f. Eq. (2.7).

## 6. Water quality in the basin water - oxygen conditions

The flux of particulate organic matter into a fjord basin is  $F_C$  ( $\text{gC month}^{-1} \text{m}^{-2}$ ). In general the flux is composed of both natural marine matter and matter directly and indirectly produced by human activities, e.g. fish farming. If  $F_C$  is known, the expected volume mean oxygen consumption in the basin water is given by

$$\frac{dO_2}{dt} = -\frac{\mu F_C}{H_b} \quad (6.1)$$

Here  $\mu$  is the amount of oxygen needed to oxidise organic matter measured as carbon. In the computations  $\mu$  is assigned the value  $3.5 \text{ gO}_2 (\text{gC})^{-1}$ . Eq. (6.1) should give the real oxygen consumption but the apparent oxygen consumption may be less because oxygen may be transported into the basin water by turbulent diffusion, see Aure and Stigebrandt (1989b). The natural part of  $F_C$ , enforced by offshore conditions, may be estimated from Eq. (2.8) with  $z=H_t$ ,  $L_m=50 \text{ m}$  and  $F_{C0}$  equal to the established regional value. In some fjords with fish farming uneaten food and faeces from the farms may contribute to the flux of organic matter into the basin water. This effect is included in FjordEnv, see below.

The time-scale  $T_O$  to reduce the oxygen concentration in the basin water by the amount  $\Delta O_2$  is

$$T_O = \frac{\Delta O_2}{\frac{dO_2}{dt}} \quad (6.2)$$

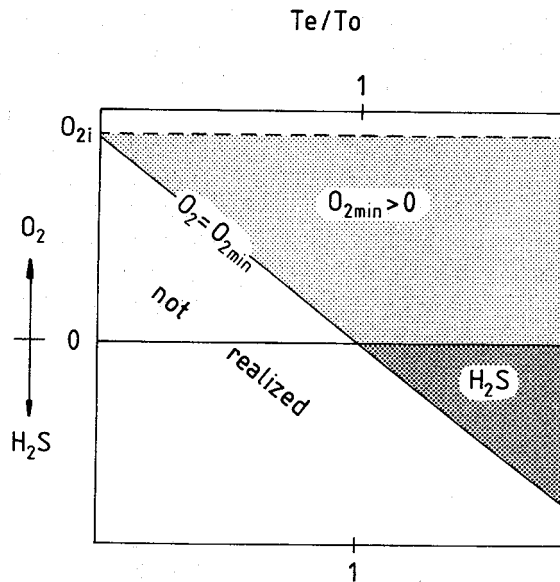


Fig. 6.1 Minimum oxygen concentration in the basin water as function of  $T_e/T_O$ .



If  $\Delta O_2$  is taken equal to the oxygen concentration of new basin water,  $O_{2in}$ , one obtains the time-scale for complete oxygen depletion. This measure usually underestimates the observed time scale because i) some oxygen is supplied to the basin water by diffusion as mentioned above and ii) as observed by Aure and Stigebrandt (1989a,b), the rate of oxygen consumption decreases when the oxygen concentration becomes lower than about 2 mlO<sub>2</sub>/l, the lowest concentration accepted by higher forms of life,

It is obvious that the lowest oxygen concentration in the basin water will occur at the end of a stagnation period. The length of stagnation periods  $T_e$  is determined by the physics, see section 4.5 If  $T_e < T_o$ , the basin water will still contain oxygen when exchanged. However, if  $T_o < T_e$  the basin water will during a stagnation period eventually be depleted in oxygen. Hydrogen sulphide will then appear, first in the deepest parts. The concentration of oxygen at the end of a stagnation period,  $O_{2min}$ , is thus dependent on both the initial oxygen concentration ( $O_{2in}$ ) and the relative lengths of  $T_o$  and  $T_e$  and is given by

$$O_{2min} = O_{2in} \left( 1 - \frac{T_e}{T_o} \right) \quad (6.3)$$

This relationship is visualised in Fig. 6.1.

In Norway it is nowadays not allowed to farm in such ways that excess food accumulates on the bottom beneath a farm. Accordingly, farms must be located to places where the benthic community continuously may assimilate excess food and faeces. Under these circumstances one may thus assume that organic matter from a farm is oxidised immediately at the seabed and figures of the kind shown in Table 3.1 may be used to compute the loading due to farming. In earlier days one had to account for delayed oxidation due to accumulation and storage of organic matter in piles beneath fish farms. The old Fjordmiljø model did this.

In chapter 4.5 it was demonstrated that the residence time for basin water  $T_e$  is inversely proportional to the intensity  $W$  of vertical mixing in the basin water, c.f. Eq. (4.21). From Eq. (6.3) above follows that a decrease of  $T_e$  by an increase in  $W$  will increase  $O_{2min}$ . This may be utilised to improve the oxygen conditions in the basin water, see also Section 6.2

### 6.1 A more complete formula for $F_c$

If the residence time  $T_t$  for water above sill depth is relatively short and the sill is not too shallow, the natural flux of organic matter into the basin water of fjords should be given by Eq. (2.9). However, there are at least three different circumstances that may invalidate the use of this equation for a particular fjord. As discussed below, these are connected to the three main potential sources of nutrients and organic matter to fjords.

The first circumstance is connected to the transport capacity of the fjord mouth. If this is small and there are no other sources of nutrients and organic matter to the fjord than those provided by the flow through the mouth, a simple budget model shows that the mean concentration of organic matter will be less in the fjord than in the coastal water. In this case the transport of organic

matter into the basin water may actually be less than that given by Eq. (2.9). To the best of the present author's knowledge there are no published examples of inshore areas of this kind.

The second circumstance is related to the coupling between regeneration of nutrients in the basin water and production of organic matter in the euphotic zone in the fjord. In fjords with sill depths appreciably greater than the depth of the euphotic zone, the regenerated nutrients in the old basin water may escape the fjord beneath the euphotic zone. In this case they may not contribute to production in the fjord. However, in fjords where the sill intrudes into the euphotic zone and the filling time of new basin water  $T_{fill}$  is long, nutrients regenerated in the basin water will spend time in the euphotic zone during the renewal of basin water. The organic matter produced by regenerated nutrients during the renewal process may settle in the basin. There are several examples of fjords of this kind. The inner Oslo Fjord should be one of them.

The third circumstance is connected to local supplies of nutrients. In fjords with shallow sills and quite long residence time  $T_t$  for water above sill level, organic matter produced by local supplies may sink deeper than the sill level in the fjord and thereby contribute to increased transport of POM into the basin waters. This is in contrast to the situation in fjords that are rapidly flushed above the sill level where the POM produced by local supplies of nutrients is exported with the surface water.

An attempt to account for these three circumstances was made by the present author who constructed the following formula for the mean vertical transport of POM into sill basins  $F_c^*$  (Stigebrandt, 1992)

$$F_c^* = F_c \left( \frac{Q}{Q + A_t w} \right) \left( 1 + \frac{v}{1-v} f_1 f_2 \right) \left( 1 + f_3 \frac{c_f}{c_1} \frac{Q_f}{Q + A_t w} \right) \quad (6.4)$$

Here  $Q = Q_f + Q_e + Q_i + Q_t$  is the rate of water exchange through the mouth,  $A_t$  the horizontal surface area of the fjord at sill depth and  $F_c$  is given by Eq. (2.9). Without local supplies of nutrients, the mean concentration of POM above the sill level of the fjord would be  $c_1$ . This may be computed from Eq. (6.4) with  $f_3=0$  using the definition  $c_1 = F_c^* / w$  where  $w$  is the mean sinking velocity of POM.  $Q_f$  is the freshwater supply to the fjord and  $c_f$  times  $Q_f$  equals the total local supply of nutrients. The fraction  $v$  of nutrients supplied by POM to the basin water is returned to the water ( $0 \leq v \leq 1$ ) while the fraction  $1-v$  is retained in the sediment. The denitrification process will give additional retention of nitrogen. The functions  $f_1$ ,  $f_2$  and  $f_3$  are briefly discussed below. For most fjords, the product  $f_1 \cdot f_2 = 0$  and  $f_3 = 0$  and  $Q / (Q + A_t w) \approx 1$  so for these fjords Eq. (6.4) gives almost the same result as Eq. (2.9).

It is thus essentially three circumstances that will lead to vertical transports POM into sill basins deviating from that predicted by Eq. (2.9). The first is due to the tendency of the mouth to choke supplies of POM and nutrients from the coastal water. This effect, that thus will reduce the transport of POM into the basin water, is described by the expression within the first pair of parentheses in Eq. (6.4). The second effect occurs in fjords where the sill intrudes into the euphotic zone (i.e.  $f_1 > 0$ ) and the water exchange is sufficiently slow (i.e.  $f_2 > 0$ ). Nutrients from the old basin water may then give rise to production of new POM in the fjord that sinks down into the basin water. This effect is described by the expression within the second pair of parentheses in Eq. (6.4). The third effect, caused by local supplies of nutrients to the surface

layer, occurs in fjords with long residence time for water above sill level (i.e.  $f_3 > 0$ ). This effect is described by the expression within the third parenthesis of Eq. (6.4).

The functions  $f_1$  and  $f_2$  are described mathematically in the following way

$$f_1 = \begin{cases} 1 & \text{if } Q/A_t w \leq 0.75 \\ 2.5 - 2Q/A_t w & \text{if } 0.75 < Q/A_t w < 1.25 \\ 0 & \text{if } Q/A_t w \geq 1.25 \end{cases} \quad (6.5)$$

$$f_2 = \begin{cases} 1 & \text{if } H_t < 15 \\ (25 - H_t)/10 & \text{if } 15 < H_t < 25 \\ 0 & \text{if } H_t \geq 25 \end{cases} \quad (6.6)$$

In Eq. (6.5)  $Q/A_t$  is the mean vertical velocity (directed upwards) at sill level during renewal of basin water.  $w$  is the mean sinking velocity of POM ( $w \approx 1.5 \text{ m day}^{-1}$ ) and  $H_t$  is the sill level.

The function  $f_1$  describes the assumed dependence on the vertical speed of the water exchange ( $Q/A_t$ ) in relation to the sinking speed  $w$  of POM. A gradual transition from one extreme to the other has been chosen (ramp function). If the vertical speed due to the exchange of basin water is less than  $0.7w$  full effect is expected, i.e. POM created by nutrients from the basin water will sink down into the basin water. If the vertical speed is greater than  $1.25w$  all new POM is assumed to be transported out of the fjord.

The function  $f_2$  describes assumed dependence of the sill depth. Also here a ramp function is used for sill depths around the depth of the euphotic zone (here taken equal to 20 m). For sill depths greater than 25 m no effect is expected because the old basin water is expected to escape the fjord beneath the euphotic zone. Full effect is expected for sill depths less than 15 m. One should note that if the exchange of basin water takes place in winter the depth of the euphotic zone may be almost vanishing but it was considered premature to include this effect in FjordEnv at the present stage of development.

One may define the sinking time for POM to sill level,  $T_p = H_t/w$ , as the time it takes for particles with the sinking speed  $w$  to sink from the sea surface to the sill level. If  $T_p$  is great compared to the residence time of water above sill level in the fjord  $T_t$ , most of POM produced in the surface layer of the fjord should be exported through the mouth, c.f. Fig. 2.3. However if  $T_p$  is shorter than  $T_t$  one should expect that some POM produced in the fjord would be exported to the basin water.

To get a reasonable transition from one extreme to the other a ramp function is used for  $f_3$

$$f_3 = \begin{cases} 0 & \text{if } T_p > 1.5T_t \\ 1.5 - T_p/T_t & \text{if } 0.5 < T_p/T_t < 1.5 \\ 1 & \text{if } T_p < 0.5T_t \end{cases} \quad (6.7)$$

If  $f_3$  equals zero, local supply of nutrients does not give rise to enhanced transport of POM into the sill basin. If  $f_3$  equals one, the local supplies of nutrients give rise to increased transport of POM into the sill basin.

The effects described above are all due to the influence of topography upon transports. In some fjords there seem to be still other effects that may give increased transports of POM into the basin water. To explain some quite high values of  $F_C$  estimated for fjords in Northern Norway, Stigebrandt et al. (1992) suggested that in very open fjords with vast shallow areas offshore, large amounts of macro algae may drift into the fjords and sink into the basin water. Also, organic matter may be eroded and transported from shallow areas of the fjord and later settle in the basin water. Another anomaly might be introduced by the pelagic ecosystem that possibly may alter the length scale  $L$  of pelagic mineralisation.

## 6.2 Specific comments concerning the FjordEnv implementation

In FjordEnv there is an option to use observational values of  $O_{2\min}$  and  $dO_2/dt$  to improve the prediction of changes of water quality in the basin water. An observationally based estimate of  $dO_2/dt$  may be used to get more accurate value of  $T_O$  and  $F_C$ . Eq. (6.3) may then be used to estimate more accurate values of  $T_e$  and  $W$  using Eq. (4.24). Using the observationally based values of  $T_O$  and  $T_e$  should improve the predictions of effects of human activities.

FjordEnv computes the extra power needed by basin water turbulence to obtain  $O_{2\min} = 2$  ml/l. How the power in practise should be added to the basin water is a technical problem not dealt with by FjordEnv, see also the last paragraph in Section 4.5.

Eq. (6.4) is implemented in FjordEnv with  $(1-\nu)=0.8$  for P.

## 7. Summary of data needed to apply the FjordEnv model

Data needed to apply a specific model are model dependent. This section describes briefly data needed to apply FjordEnv. Topographic data are needed to compute volumes of different depth strata in the fjord and the vertical cross-sectional area of the mouth. The user of FjordEnv has to find data on the surface area of the fjord and the width of the mouth, respectively, at different depths. Usually it is possible to extract this information from sea charts.

When running FjordEnv one has to tell the program in which region the inshore area is located. This is because FjordEnv has stored data relevant for the regional forcing of water exchange and the vertical flux of organic matter. Among forcing data are the amplitude of the semidiurnal tide ( $M_2+S_2$ ) and the fractional contribution of the semidiurnal tide to the total barotropic to baroclinic energy transfer at sills ( $1/\phi$ ). The variability of the offshore density field ( $\sigma_\rho(z)$ ) is used to compute a number of quantities in FjordEnv. Among these are the forcing of intermediary water exchange ( $\Delta M$ ), speed of the first internal wave mode with a pycnocline at sill depth ( $c_i$ ) and the density reduction needed in a sill basin before water exchange takes place ( $R_e$ ). Furthermore, the flux of POM into the sill basin ( $F_C$ ) is obtained with help of the database.

The user of FjordEnv should know the local freshwater supply and local supplies of nutrients and organic matter by fish farming and other outlets due to human activities in and around the inshore area under consideration, see Section 3. This information may often be obtained from official authorities.

To compute changes in Secchi depth and oxygen conditions in the basin water due to changes in supplies of nutrients and organic matter, the Secchi depth before the changes should be given as well as the oxygen concentration in new basin water. Changes in supplies of N and P and fish production are given to the program.

If the inshore area is composed of coupled basins, sills between the basins may cause additional mixing in the outer basin. This may be handled in FjordEnv by running the model on each basin, starting from the innermost basin.

The computation of the conditions in the basin water may be improved if observational data on the rate of oxygen consumption and oxygen minimum in the basin water are supplied to the model (optional).

## 8. Concluding remarks

The water quality FjordEnv was primarily developed for fjords but is applicable also to other types of inshore areas. FjordEnv has been applied to Norwegian fjords during about one decade. Since the model computes the different modes of water exchange using mechanistic models, it should be applicable to inshore areas also in other coastal regions of the world. However, to extend the applicability of the model to new regions one first has to compile offshore data to extract the remote forcing of water exchange and the natural flux of organic matter.

There are certainly a number of possible improvements of FjordEnv but only a few examples are given here. In the expression for the Secchi depth as function of POP (Eq. (2.7)), the regional values of  $k_b$  should also be given but this is not implemented so far. The suggested improvement of the formula for the vertical flux of organic matter into the sill basin presented in Eq. (6.4) is quite uncertain and should be regarded as a first attempt. Among other things the functions  $f_1$ ,  $f_2$ ,  $f_3$  are known only in a rudimentary way and should be investigated further.

One of the main ideas behind FjordEnv is that the model should be simple to use. This means that the burden laid upon the user to find data and parameter values should be kept as small as possible. Therefore, improvements of the model should essentially concern improvements of process descriptions.

FjordEnv computes environmental effects horizontally averaged over the whole inshore area. A specific model has been developed to compute the local environmental conditions in fish farms. This model, MOM (Stigebrandt and Aure, 1995; Ervik et al. 1997; Stigebrandt et al. 2001), computes the holding capacity of a site. This is estimated based on three different requirements; (i) the benthic fauna is not allowed to disappear due to accumulation of organic material under the farm; (ii) there must be good living conditions for the fish in the farm; (iii) the farm must not appreciably deteriorate the water quality in the areas surrounding the farm. The lowest of the estimates defines the holding capacity. Estimates (i) and (ii) are computed using the MOM model and estimate (iii) is computed using FjordEnv.

## 9. Acknowledgements

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## Appendix: List of symbols

$a_i$	inshore tidal amplitude
$a_0$	offshore tidal amplitude
$A_f$	horizontal surface area of the inshore area at the sea surface ( $\text{km}^2$ )
$A_t$	horizontal surface area of the inshore area at sill depth ( $\text{km}^2$ )
$A_m$	vertical cross-sectional area of the mouth ( $\text{m}^2$ )
$B_m$	width of the mouth
$cc$	choking coefficient
$C$	Carbon
$C_e$	constant in the model of estuarine circulation
$c_i$	speed of internal wave
$c_{Pf}$	change of concentration of P ( $\text{mmol P m}^{-3}$ )
$C_D$	drag coefficient
$C_S$	empirical constant (Eq. 2.3)
$C_W$	empirical constant
$D_f$	depth of euphotic zone (m)
$D_S$	Secchi depth (m)
$D_0$	Secchi depth, before changes in nutrient supply (m)
$E_j, E_2$	energy transfer to internal tides at sills
$E_{jet}$	energy transfer to a tidal jet
$f$	Coriolis parameter
$f_1$	function defined in Eq. (6.5)
$f_2$	function defined in Eq. (6.6)
$f_3$	function defined in Eq. (6.7)
$F_C(z)$	vertical flux of carbon at depth $z$ ( $\text{g C month}^{-1}\text{m}^{-2}$ )
$F_{C0}$	vertical flux of carbon close to the sea surface ( $\text{g C month}^{-1}\text{m}^{-2}$ )
$g$	acceleration of gravity ( $9.82 \text{ m s}^{-2}$ )
$g' = g\Delta\rho/\rho_0$	buoyancy ( $\text{m s}^{-2}$ )
$H_t$	sill depth (m)
$H_b$	mean depth of the sill basin (m)
$H_{1m}$	thickness of upper layer (m) (2-layer approx.)
$H_{2m}$	thickness of lower layer (m) (2-layer approx.)
$I(z)$	light intensity at depth $z$ ( $\text{Wm}^{-2}$ )
$I_0$	light intensity at sea surface ( $\text{Wm}^{-2}$ )
$k_d$	vertical attenuation coefficient ( $\text{m}^{-1}$ )
$k_p$	vertical attenuation coefficient, contribution by phytoplankton ( $\text{m}^{-1}$ )

$k_b$	vertical attenuation coefficient, “background” contribution ( $m^{-1}$ )
$L_m$	length scale of pelagic mineralisation (m)
$\Delta M$	forcing of intermediary circulation ( $kg\ m^{-2}$ )
$m_0$	efficiency factor of turbulence
N	Nitrogen
$\Delta O_2$	reduction of oxygen concentration ( $g\ O_2\ m^{-3}$ )
$O_{2min}$	oxygen minimum in the basin water ( $g\ O_2\ m^{-3}$ )
$O_{2in}$	oxygen concentration in new basin water ( $g\ O_2\ m^{-3}$ )
P	Phosphorus
$P_f$	changed supply of P ( $mmol\ P\ s^{-1}$ )
POM	particulate organic matter
POP	phosphorus in suspended particulate organic matter ( $mmol\ P\ m^{-3}$ )
Q	flow ( $m^3\ s^{-1}$ )
$Q_{bc}$	baroclinic geostrophic flow ( $m^3\ s^{-1}$ )
$Q_f$	freshwater supply ( $m^3\ s^{-1}$ )
$Q_1$	outflow from surface layer ( $m^3\ s^{-1}$ ) (2-layer approx.)
$Q_2$	inflow in lower layer ( $m^3\ s^{-1}$ ) (2-layer approx.)
$Q_e=Q_2$	estuarine circulation ( $m^3\ s^{-1}$ )
$Q_i$	intermediary water exchange ( $m^3\ s^{-1}$ )
$Q_s$	water exchange of a s m thick surface layer ( $m^3\ s^{-1}$ )
$Q_t$	tidally forced water exchange ( $m^3\ s^{-1}$ )
Rf	Richardson flux number
$R_e$	density reduction between events of basin water exchange ( $kg\ m^{-3}$ )
S	salinity
$S_1$	salinity of upper layer (2-layer approx.)
$S_2$	salinity of lower layer (2-layer approx.)
t	time
T	Time scale
$T_m$	time-scale for mineralisation
$T_f$	residence time of water in the surface layer
$T_e$	time-scale for exchange of basin water
$T_{fill}$	filling time for exchange of basin water
$T_O$	time-scale for oxygen consumption in the basin water
$T_p$	time-scale for particles to sink the vertical distance $H_t$
$T_t$	time-scale for exchange of water above sill level
$u_{s0}$	tidal current amplitude in the mouth
$u_{1m}$	upper layer current speed in the mouth (2-layer)

$u_{2m}$	lower layer current speed in the mouth (2-layer)
$u_*$	friction velocity ( $m s^{-1}$ )
$V$	total volume of the inshore area
$V_t$	volume above sill level
$V_b$	volume below sill level
$V_s$	volume of a $s$ meter thick surface layer
$W$	wind speed ( $m s^{-1}$ )
$W_e$	entrainment velocity ( $m s^{-1}$ )
$w$	sinking speed of particulate matter ( $m day^{-1}$ )
$W$	mean rate of work against buoyancy forces ( $W m^{-2}$ )
$W_0$	mean rate of “background” work against buoyancy forces ( $W m^{-2}$ )
$z$	depth
$\beta$	salt contraction coefficient of sea water
$\gamma$	empirical constant (intermediary circulation)
$\varepsilon$	efficiency factor of tidally forced water exchange
$\phi$	layer thickness ratio (model of estuarine circulation)
$\Delta\eta$	sea level difference (m)
$\Delta\rho$	density difference ( $kg m^{-3}$ )
$\rho$	density ( $kg m^{-3}$ )
$\rho_0$	reference density ( $kg m^{-3}$ )
$\rho_f$	density of freshwater ( $kg m^{-3}$ )
$\rho_b$	mean density of the basin water ( $kg m^{-3}$ )
$\rho_t$	mean density of water above the sill level ( $kg m^{-3}$ )
$\mu$	oxygen to carbon ratio for oxidation of organic matter
$1-\nu$	retention of nutrients
$\sigma_\rho$	standard deviation of offshore density
$\omega$	frequency

## **OARRE**

### **Oceanographic Applications to Eutrophication in Regions of Restricted Exchange**

OAERRE aims to understand the physical, biogeochemical and biological processes, and their interactions, involved in eutrophication in coastal marine Regions of Restricted Exchange (RREs), especially lagoons and fjords. The scientific issues addressed include the controls on horizontal and vertical exchange in RREs and the response of coastal ecosystems to nutrient enrichment.

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MARE – Marine Research on Eutrophication: A scientific base for cost-effective measures for the Baltic Sea – is a four-year, multidisciplinary research programme financed by the Swedish Foundation for Strategic Environmental Research, MISTRA. The overall goal of MARE is to develop a decision support system to be used as a tool for developing and testing cost-effective strategies to reduce eutrophication in the Baltic Sea.

MARE  
c/o Swedish Environmental Protection Agency  
SE- 106 48 Stockholm, Sweden  
Tel. +46 8 698 1536  
Fax. +46 8 698 1664  
Web site: [www.mare.su.se](http://www.mare.su.se)